t-test

- Given by W.S. Gosset in 1908 under the pen name of student's test
- t-test can be applied when:

1. When a researcher draws a small random sample ($n<30$) to estimate the population (μ);
2. When the population standard deviation (σ) is unknown;
3. The population is normally distributed

Q: Royal tyre has launched a new brand of tyres for tractors & claims that under normal circumstances the average life of tyres is 40000 km. a retailer wants to test this claim & has taken a random sample of 8 tyres. He tests the life of tyres under normal circumstances. The results obtained are:

Tyres	1	2	3	4	5	6	7	8
Km	35 000	38 000	42 000	41 000	39 000	41 500	43 000	38 500

Use $\alpha = 0.05$ for testing the hypothesis

Step1: Set null & alternative hypothesis

Null hypothesis: Ho: $\mu = 40\,000$

Alternative hypothesis: Ho: $\mu \neq 40\,000$

Step2: Determine the appropriate statistical test

The sample size is less than 30, so t test will be an appropriate test

Step3: Set the level of significance

The level of significance, i.e. $\alpha = 0.05$

Step4: Set the decision rule

The t distribution value for a two-tailed test is $t_{0.025} = 2.365$ for degrees of freedom 7.

so if computed t value is outside the $\pm$ 2.365 range, the null hypothesis will be rejected otherwise accepted.

- Step 5: Collect the sample data:

Tyres	1	2	3	4	5	6	7	8
Km	350000	38000	42000	41000	39000	41500	43000	38500

- Step 6: Analyze the data

X=39750; μ=40000; s=2618.61 n=8; df=n-1=7 ;

Table value of $t_{0.025,7}$=2.365

- $t = \dfrac{x - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{39750-40000}{\frac{2618.61}{\sqrt{8}}} = -0.27$

- Step 7: Arrive at a statistical conclusion & Business implication

The observed t value is -0.27 which falls within the acceptance region & hence null hypothesis is accepted i.e. Ho: μ = 40 000

Z-test

- Hypothesis testing for large samples i.e. $n >= 30$;
- Based on the assumption that the population , from which the sample is drawn, has a normal distribution;
- As a result, the sampling distribution of mean is also normally distributed;

Application:

1. For testing hypothesis about a single population mean;
2. Hypothesis testing for the difference between two population means;
3. Hypothesis testing for attributes.

Formula for single population mean (finite population)

- $$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Where,

μ = population mean

$\bar{x}$ = sample mean

σ = population standard deviation

n = sample size

Q A marketing research firm conducted a survey 10 yrs ago & found that an average household income of a particular geographic is Rs 10000. Mr. gupta who recently joined the firm a VP expresses doubts. For verifying the data, firm decides to take a random sample of 200 households that yield a sample mean of Rs 11000. assume that the population S.D is Rs 1200. verify Mr. Gupta's doubts using $\alpha=0.05$?

- Step 1: set null & alternative hypothesis

Ho: $\mu=10000$

H1: $\mu\neq10000$

- Step2: Determine the appropriate statistical test

Since sample size >=30, so z-test can be used for hypothesis testing

- Step3: set the level of significance

The level of significance is known ($\alpha=0.05$)

- Step4: Set the decision rule

Acceptance region covers 95% of the area & rejection region 5%

Critical area can be calculated from the table (± 1.96)

- <u>Step5: collect the sample data</u>

A sample of 200 respondents yield a sample mean of Rs 11000

- <u>Step6: Analyze the data</u>

$n=200$ $\mu=10\ 000$

$\bar{x}=11000$ $\sigma=1200$

- $$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{11000-10000}{\frac{1200}{\sqrt{200}}} = 11.79$$

- <u>Step7: Arrive at a statistical conclusion & business implication</u>

Z value is 11.79 which is greater than +1.96, hence null hypothesis is rejected and alternative hypothesis is accepted. Hence Mr. Gupta's doubt about household income was right.